

B. TECH.
(SEM. I) THEORY EXAMINATION 2018-19
ENGINEERING MATHEMATICS -I

Time: 3 Hours

Total Marks: 100

Note: Attempt the questions in any order and then choose suitably.

SECTION - A

1. Attempt **ALL** the parts: (2x10 = 20)
- (a) Find y' , if $y = \sin x$ at $x = 0$.
- (b) If $u = x, y = \sqrt{x} + \bar{y}$, find the value of $x \frac{\partial}{\partial x} + 2xy \frac{\partial}{\partial y} + y \frac{\partial}{\partial z}$.
- (c) Calculate $\frac{d}{dt}$ for $x = e \cos v$, and $y = e \sin v$.
- (d) Prove that $f(ax) = f(x) + a - 1 \cdot xf'(x) + \frac{a^2 - 1}{2!} f''(x) + \frac{a^3 - 3a + 2}{3!} f'''(x) + \dots$
- (e) Reduce the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ into normal form.
- (f) Find the inverse of the matrix $A = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$.
- (g) Find the value of $\int x e^{-\sqrt{x}} dx$.
- (h) Evaluate $\int x e^{xy} dy dx$.
- (i) Show that $\vec{F} = x \hat{i} - y \hat{j} + x \hat{k} - 2xy \hat{j} + y \hat{k}$ is irrotational.
- (j) State Gauss divergence theorem.

SECTION - B

2. Attempt any **THREE** of the following: (10x3=30)
- (a) If $y = \sin a \sin x$, show that $(1 - x^2) y'' - 2x y' + (a^2 - 1) y = 0$ and calculate $y(0)$.
- (b) Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{6} + \frac{y^2}{3} + \frac{z^2}{2} = 1$.
- (c) Diagonalise the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$.
- (d) Find the volume of the solid surrounded by the surface $\frac{x^2}{6} + \frac{y^2}{3} + \frac{z^2}{2} = 1$.
- (e) Verify Stoke's theorem for $\vec{F} = x \hat{i} + xy \hat{j}$ integrated round the square whose sides are $x = 0, y = 0, x = a, y = a$ in the plane $z = 0$.

SECTION – C

3. Attempt any **TWO** of the following: (5x2=10)
- (a) Verify Euler's theorem for the function: $u = \dots$
- (b) If $u = f(y-z), z = x, x = y$, prove that $\dots + \dots + \dots = 0$.
- (c) Trace the curve: $y = x$.
4. Attempt any **TWO** of the following: (5x2=10)
- (a) Expand $\tan^{-1} x$ in powers of $x-1$ and $y-1$ upto two degree terms.
- (b) Show that $u = \dots, v = \dots, w = \dots$ are not independent, find the relation among them.
- (c) Find the extreme values of $3x^2 - y + x$.
5. Attempt any **TWO** of the following: (5x2=10)
- (a) Show that the system of equations:
 $x + 3y - 2z = 0, 2x - y + 4z = 0, x - 11y + 14z = 0$ has a non-trivial solution.
- (b) Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ and hence find A^{-1} .
- (c) Prove that the each characteristic roots of a unitary matrix are of unit modulus.
6. Attempt any **TWO** of the following: (5x2=10)
- (a) Evaluate $\int \dots$ by changing the order of integration.
- (b) Prove that $\sqrt{\dots} = \sqrt{\dots} B, \dots$.
- (c) Show that in the Catenary $y = c \cosh \frac{x}{c}$, the length of arc from the vertex $x = 0$ to any point x, y is given by $s = c \sinh \frac{x}{c}$.
7. Attempt any **TWO** of the following: (5x2=10)
- (a) Find the directional derivative of $\phi = 5x^2y - 5yz + z^2x$ at the point $P(1,1,1)$ in the direction of the line $\frac{x}{2} = \frac{y}{-1} = \frac{z}{-1}$.
- (b) Prove that $\text{div } \vec{a} \times \vec{b} = \vec{b} \cdot \text{curl } \vec{a} - \vec{a} \cdot \text{curl } \vec{b}$
- (c) Apply Green's theorem to evaluate $\int_C (2x^2 - y) dx + (x^2 + y) dy$ where C is the boundary of the area enclosed by the x -axis and upper half of the circle $x^2 + y^2 = a^2$.
